Robust SOLT and Alternative Calibrations for Four-Sampler Vector Network Analyzers

Jeffrey A. Jargon, *Member, IEEE*, Roger B. Marks, *Senior Member, IEEE*, and Doug K. Rytting

***Abstract*—This paper assesses the accuracy of several proposed lumped-element calibration techniques for four-sampler vector network analyzers in the face of imperfectly defined standards. We find that these methods offer a wide range of accuracies, depending on the standards and how well they can be modeled. We introduce a new robust short-open-load-thru calibration that requires the same measurement and computational effort as traditional methods, but provides much better accuracy in transmission measurements and slightly better accuracy in reflection measurements.**

***Index Terms*—Alternative, calibration, lumped element, short-open-load-thru, vector network analyzer.**

I. Introduction

THE original applications of the short-open-load-thru (SOLT) calibration to vector network analyzers (VNA's) presumed a three-sampler architecture. The method is commonly applied even to four-sampler VNA's, with the data available from the fourth sampler simply ignored. Early in this decade, two papers considered variations of the SOLT method that presumed switch-corrected data as input [1], [2] and were, therefore, appropriate to four-sampler VNA's. The number of measurements in the calibration can be less than in conventional SOLT if we make use of the data from the fourth sampler. In principle, many options are possible.

As noted by [2],

> ...the methods are not equally sensitive to measurement errors and calibration standard accuracy ... thus, measurement errors and error progression mainly depend on the quality of the test equipment and the standards used. More detailed investigations will have to be undertaken in this field.

Such an investigation is the subject of this paper. The results are essential if VNA users are to have confidence in some of the faster calibration methods.

Instead of reducing the number of standards, we can try to use the data from the fourth sampler to improve the accuracy of the calibration, making it more robust with respect to errors in the definitions of the standards. Using the results shown below, we will introduce and evaluate a new robust SOLT method based on this approach.

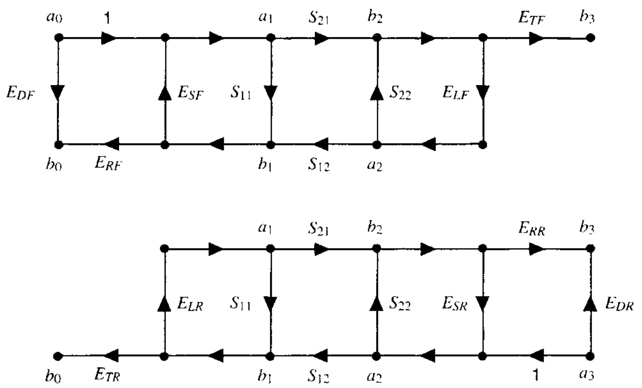

Fig. 1. Twelve-term error model of a VNA (less crosstalk terms).

II. Four-Sampler VNA's

Many advanced VNA's use a four-sampler architecture. This allows the scattering parameters of a two-port device (S_{ij} for $i, j = 1, 2$) to be determined by measuring the incident and reflected waves at each port.

Most commercial four-sampler VNA's are actually three- and four-sampler hybrids since they can be built less expensively. These hybrid VNA's are capable of measuring the fourth wave, but normally measure only three. During three-sampler operation, two of the samplers are always connected to measure the reflected waves. The third is switched to measure the incident wave on the port at which the signal is applied. Due to the effect of switching on the three-sampler system, we must represent the measurement system separately in the forward and reverse cases. We must also take into account the nature of the load that terminates the unstimulated port. Usually, the same termination is used in both forward and reverse modes, but because of the different connection paths, it is not accurate to model them identically. Fig. 1 illustrates a common representation of the VNA error model, widely known as the 12-term error model. Only ten terms are shown in Fig. 1; the crosstalk terms are left out.

Fig. 2 shows an alternative "error-box formulation" of the error model [3]. This formulation offers several advantages over the common 12-term representation: it is more directly related to the physical model of the VNA; it allows a more straightforward adaptation of calibration methods designed for four-sampler VNA's, such as thru-reflect-line (TRL) and line-reflect-match (LRM); and it is better suited to recomputing a

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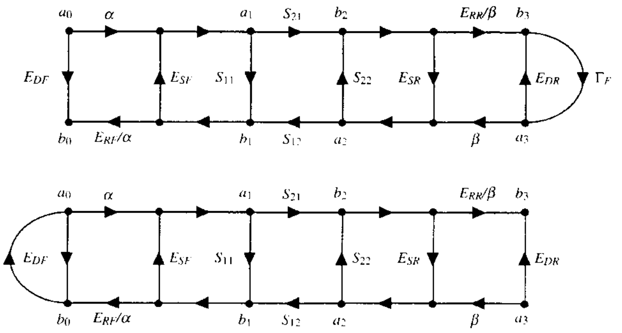

Fig. 2. Error-box error model of a VNA.

set of error coefficients when adaptors or cables are added to or removed from the reference plane.

Two "switch terms" (Γ_F and Γ_R) are introduced to account for the change of port reflection as the switch is thrown. On a four-sampler VNA, they can easily be measured and stored so that subsequent measurements can be obtained in three-sampler mode. The variations of the SOLT method that we consider in this paper make use of switch-corrected data as input.

It is possible to relate the parameters of the 12-term and error-box formulations. Six terms (E_{DF}, E_{RF}, E_{SF}, E_{DR}, E_{RR}, and E_{SR}) are equivalent in the two cases. The four remaining parameters in the 12-term case, (E_{TF}, E_{LF}, E_{TR}, and E_{LR}), can be related to the four remaining parameters of the error-box case (α, β, Γ_F, and Γ_R) by measuring a direct connection of ports 1 and 2. This yields [3]

$$\Gamma_F = \frac{E_{LF} - E_{SR}}{E_{RR} + E_{DR}(E_{LF} - E_{SR})} \tag{1}$$

$$\Gamma_R = \frac{E_{LR} - E_{SF}}{E_{RF} + E_{DF}(E_{LR} - E_{SF})} \tag{2}$$

$$\frac{\alpha}{\beta} = \frac{E_{TF}\Gamma_F}{E_{LF} - E_{SR}} = \frac{E_{TF}}{E_{RR} + E_{DR}(E_{LF} - E_{SR})} \tag{3}$$

and

$$\frac{\beta}{\alpha} = \frac{E_{TR}\Gamma_R}{E_{LR} - E_{SF}} = \frac{E_{TR}}{E_{RF} + E_{DF}(E_{LR} - E_{SF})}. \tag{4}$$

Obviously, (3) and (4) are consistent only if

$$E_{TF}E_{TR} = \frac{(E_{LF} - E_{SR})(E_{LR} - E_{SF})}{\Gamma_R\Gamma_F} \tag{5}$$

which can also be expressed solely in terms of the 12-term model parameters as

$$E_{TF}E_{TR} = \Big[E_{RR} + E_{DR}(E_{LF} - E_{SR})\Big] \cdot \Big[E_{RF} + E_{DF}(E_{LR} - E_{SF})\Big]. \tag{6}$$

For completeness, the equations for computing the 12-term parameters from the error-box parameters are

$$E_{LR} = E_{SR} + \frac{E_{RR}\Gamma_F}{1 - E_{DR}\Gamma_F} \tag{7}$$

$$E_{LR} = E_{SF} + \frac{E_{RF}\Gamma_R}{1 - E_{DF}\Gamma_R} \tag{8}$$

$$E_{TF} = \frac{E_{LF} - E_{SR}}{\Gamma_F}\,\frac{\alpha}{\beta} \tag{9}$$

TABLE I
LUMPED-ELEMENT CALIBRATION COMBINATIONS

Port 1	Port 2	Designation	Category
O S L	O S L		
1 1 1	0 0 0	$O_1S_1L_1T$	A
1 1 0	0 0 1	$O_1S_1L_2T$	B
1 0 1	0 1 0	$O_1L_1S_2T$	
0 1 1	1 0 0	$S_1L_1O_2T$	
1 1 0	0 1 0	$O_1S_1S_2T$	C
1 1 0	1 0 0	$O_1S_1O_2T$	
1 0 1	0 0 1	$O_1L_1L_2T$	D
0 1 1	0 0 1	$S_1L_1L_2T$	
1 0 1	1 0 0	$O_1L_1O_2T$	E
0 1 1	0 1 0	$S_1L_1S_2T$	

and

$$E_{TR} = \frac{E_{LR} - E_{SF}}{\Gamma_R}\,\frac{\beta}{\alpha}. \tag{10}$$

III. SOLT AND ITS VARIATIONS

The SOLT calibration (here using the algorithm of [4]) makes use of a "thru" connection of the two VNA ports as well as the measurement (on both ports) of three one-port standards, typically a nominal open, nominal short, and nominally matched load. None of these needs to be ideal, but we must know their reflection coefficients. In practice, our "definition" of those values is typically drawn from a model of the standard.

One variation of SOLT for four-sampler VNA's [1] has been dubbed "QSOLT." The "Q" (for "quick") signifies that the method is faster than SOLT since the three one-ports need be connected to only one of the VNA ports. Reference [2] included a number of variations in which the one-ports were connected to only one of the two VNA ports or, in some cases, not measured at all.

Although both [1] and [2] demonstrated the basic functionality of their proposals, neither studied the robustness of the proposed methods in the face of the inevitable discrepancy between the reflection coefficients of the standards and our definition of those values.

In addition to QSOLT, Table I lists the nine other combinations for which two standards are connected to Port 1 and one to Port 2. An additional ten combinations are possible by swapping the ports. In the table, each calibration is given a designation, which refers to the standards and the ports to which they are connected.

In Table I, we have categorized the calibrations as follows:

- Category A: three unique standards measured on Port 1 (QSOLT);
- Category B: three unique standards, one of which is measured on Port 2;
- Category C: open and short measured on Port 1; one remeasured on Port 2.; no load;
- Category D: load measured on both ports, open or short measured on Port 1;

Category E: open or short measured on both ports, load measured on Port 1.

IV. The Simulator

Our accuracy study makes use of a measurement simulator that simulates "raw" VNA output from input that represents the actual scattering parameters of several physical standards and test devices. After calibrating with the raw measurements of the standards, we apply error correction schemes to raw data for the test devices. Since we have access to the true scattering parameters of each test device, we can explicitly determine the error introduced by each calibration. Other approaches that simply compare the measurement results produced by two calibrations cannot determine the accuracy of either or even say definitively which is better.

In our studies, the input data were measured on a VNA calibrated using multiline TRL carried out with MultiCal software [5]. We provide to the simulator the VNA calibration coefficients determined in that process; with these, it simulates the raw VNA measurements. The simulator operates on calibrated data for a thru and for pairs of nominal opens, shorts, and loads. In addition, we include measurements of a 19-mm transmission line, which serves as a device-under-test (DUT). All of the measured devices were implemented in coplanar waveguide on GaAs.

V. Calibration

Several published mathematical procedures [2], [6], [7] allow calibration using the measurements described in Table I. Here, we made use of an alternative formulation of the "error-box model," in which we created a 7×7 complex linear system [4]. One set of four equations uses the switch-corrected measurements of the scattering parameters of the thru as follows:

$$E_{DF} + S_{11}S_{11M}E_{SF} + S_{11}\Delta_F + 0 + S_{21}S_{12M}(\alpha/\beta)E_{SR} + 0 + 0 = S_{11M} \quad (11a)$$

$$0 + S_{12}S_{11M}E_{SF} + S_{12}\Delta_F + 0 + S_{22}S_{12M}(\alpha/\beta)E_{SR} + 0 - S_{12M}(\alpha/\beta) = 0 \quad (11b)$$

$$0 + S_{11}S_{21M}E_{SF} + 0 + 0 + S_{21}S_{22M}(\alpha/\beta)E_{SR} + S_{21}(\alpha/\beta)\Delta_R + 0 = S_{21M} \quad (11c)$$

$$0 + S_{12}S_{21M}E_{SF} + 0 + (\alpha/\beta)E_{DR} + S_{22}S_{22M}(\alpha/\beta)E_{SR} + S_{22}(\alpha/\beta)\Delta_R - S_{22M}(\alpha/\beta) = 0 \quad (11d)$$

where

$$\Delta_F = E_{RF} - E_{SF}E_{DF} \quad \Delta_R = E_{RR} - E_{SR}E_{DR}. \quad (12)$$

Three more equations of the form (11a) or (11d), depending on which port is used for the measurements, are determined by the switch-corrected reflection-coefficient measurements of the three one-port standards. Solution of the linear system yields the seven error coefficients that describe the four-sampler VNA (ignoring crosstalk errors). We also tried an alternative algorithm [1], but saw no significant difference.

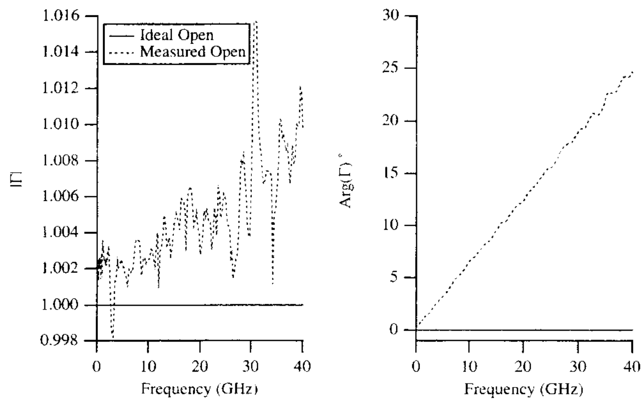

Fig. 3. Corrected reflection coefficient of the measured open circuit.

VI. Accuracy

Construction of the 7×7 system requires the "true" reflection coefficients Γ of the standards. When the simulator used these data, all of the calibration procedures proved functional.

However, we normally have no access to this information in practice. Instead, we need to model the standards. The essence of this study is to determine the sensitivity of the calibration methods to errors in these models. We tested the calibrations in the following three cases:

1) simplistic model of the open ($\Gamma = +1$ for the open);
2) simplistic model of the short ($\Gamma = -1$ for the short);
3) simplistic model of the load ($G = 0$ for the load).

In each case, we used the "true" Γ of the other two lumped-element standards. As an example, Fig. 3 illustrates the actual Γ of the open, along with its simplistic model. We offset the open to illustrate the effect of nonideal phase.

In the case of Category A (QSOLT), the effect of using a simplistic model was qualitatively similar for any standard. Fig. 4 illustrates the effect of the simplistic open model on QSOLT and SOLT. QSOLT provides significantly improved accuracy in transmission measurement with respect to SOLT, with a slight accuracy gain in S_{11}. However, QSOLT could not accurately obtain S_{22}. This performance can be explained by the fact the QSOLT uses no standards on Port 2. However, the result is not apparent from prior publications. Reference [1] suggested that QSOLT appeared to provide somewhat better accuracy than SOLT in S_{12} and S_{21} and "seems to be reasonably better" for S_{22}. This can be explained by the limited data available. Reference [2] did not show data for S_{22}. Its transmission data were not compared to other measurements and were inconclusive.

In the Category B calibrations, one of the three unique standards is measured on Port 2. When one of the standards was simplistically modeled, the Category B calibrations still provided improved transmission accuracy with respect to SOLT. A simplistic load model gave reflection results comparable to SOLT. With a simplistically modeled reflect (i.e., open or short) instead, S_{11} was comparable to SOLT and S_{22} was less accurate if the load was on Port 1, whereas the opposite was true if the load was on Port 2. Fig. 5 illustrates the effect of the simplistic open model on $O_1S_1L_2T$ and SOLT.

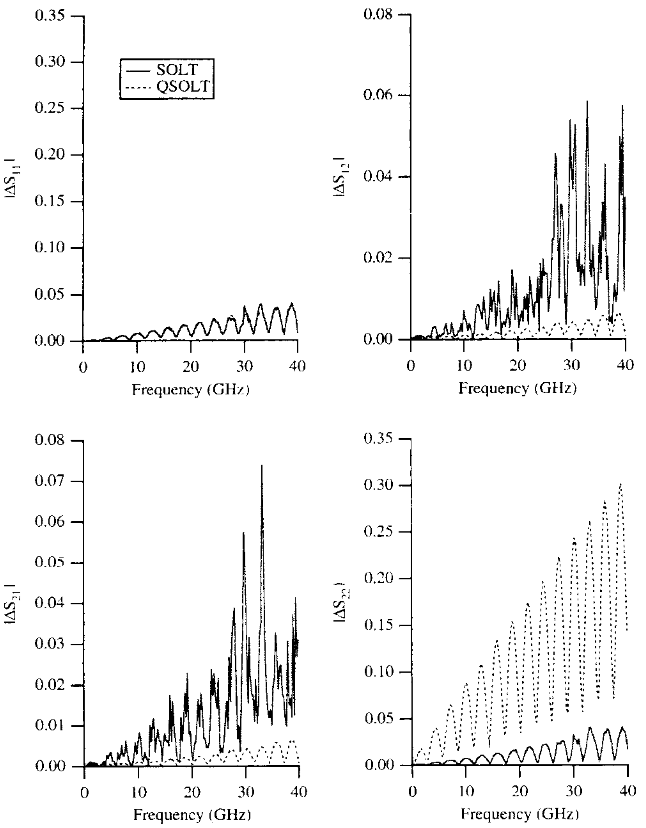

Fig. 4. Magnitude of S-parameter errors ($|\Delta S_{ij}|$) using SOLT and QSOLT (Category A) to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model. QSOLT standards are measured on Port 1 only.

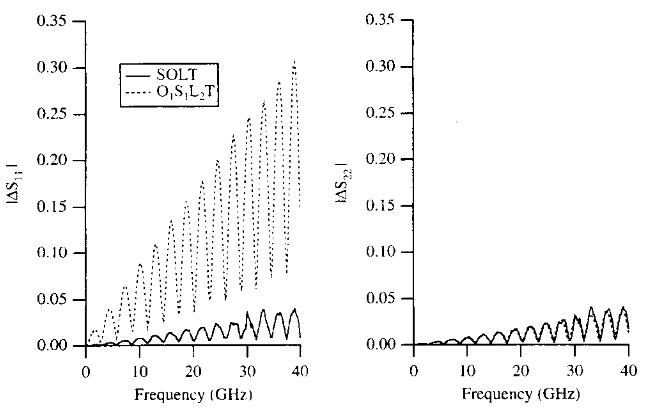

Fig. 5. Magnitude of reflection-coefficient errors ($|\Delta S_{11}|$ and $|\Delta S_{22}|$) using SOLT and $O_1S_1L_2T$ (Category B) to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model.

$O_1S_1L_2T$ provides a slight accuracy gain in S_{22} with respect to SOLT since the load is measured on Port 2, but it could not obtain S_{11} accurately without a load measurement on Port 1.

As expected, Category C, which did not include a load, performed poorly. When the reflect measured on one port was simplistically modeled, the Category C calibrations provided better transmission accuracy than SOLT, although with very poor reflection accuracy. Fig. 6 illustrates the effect of the simplistic open model on $O_1S_1S_2T$ and SOLT. $O_1S_1S_2T$ could not correctly obtain either S_{11} or S_{22} without a load measurement. When the reflect measured on both ports was simplistically modeled, the results were disastrous for all scattering parameters, presumably because that standard is used twice.

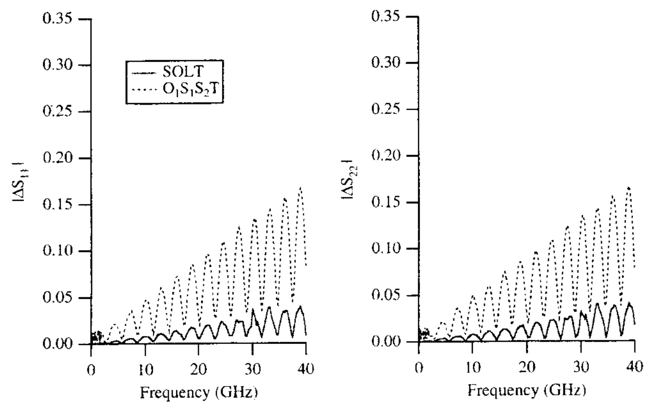

Fig. 6. Magnitude of reflection-coefficient errors ($|\Delta S_{11}|$ and $|\Delta S_{22}|$) using SOLT and $O_1S_1S_2T$ (Category C) to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model.

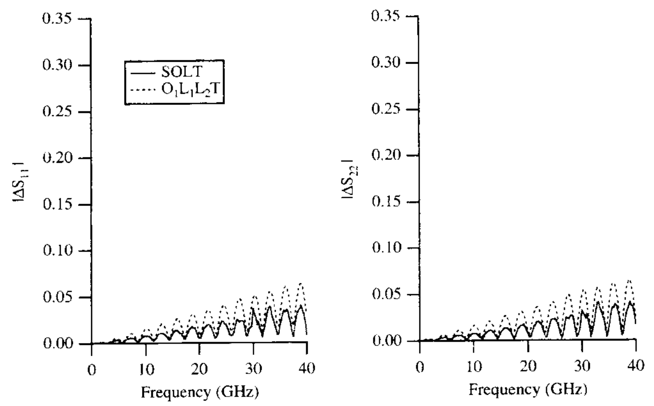

Fig. 7. Magnitude of reflection-coefficient errors ($|\Delta S_{11}|$ and $|\Delta S_{22}|$) using SOLT and $O_1L_1L_2T$ (Category D) to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model.

Clearly, lumped-element calibrations will fail when all of the standards have reflection coefficients of $+1$ or -1 because these reflection coefficients are invariant to reference impedance. To fully understand the capabilities of Category C calibrations, we would need to study their performance using reflects that avoid these two critical points (e.g., offset opens and shorts). The resulting reflection coefficients would be more difficult to model. The advantage of the Category C calibrations, however, is that they do not require a load and are, therefore, free of errors due to inaccuracy in the load definition.

Category D calibrations ignore either the open or short, but measure the load on both ports. This may be advantageous if only one well-characterized high-reflection standard is available. A simplistic load model gave reflection results comparable to SOLT, but better transmission results. With a simplistically modeled reflect instead, measured transmission was more accurate, but the reflection was slightly less accurate. Fig. 7 illustrates the effect of the simplistic open model on $O_1L_1L_2T$ and SOLT. $O_1L_1L_2T$ obtained both S_{11} and S_{22}

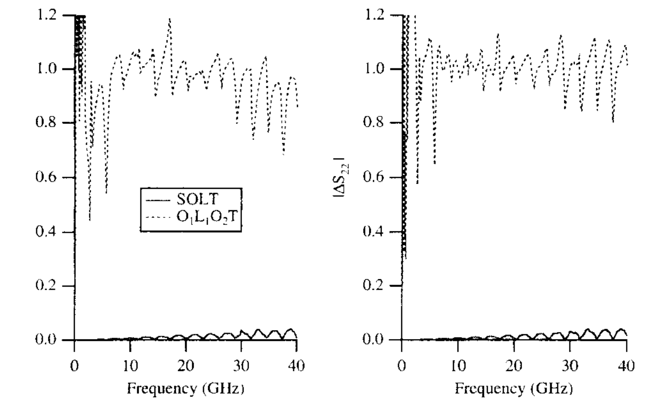

Fig. 8. Magnitude of reflection-coefficient errors ($|\Delta S_{11}|$ and $|\Delta S_{22}|$) using SOLT and $O_1L_1O_2T$ (Category E) to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model.

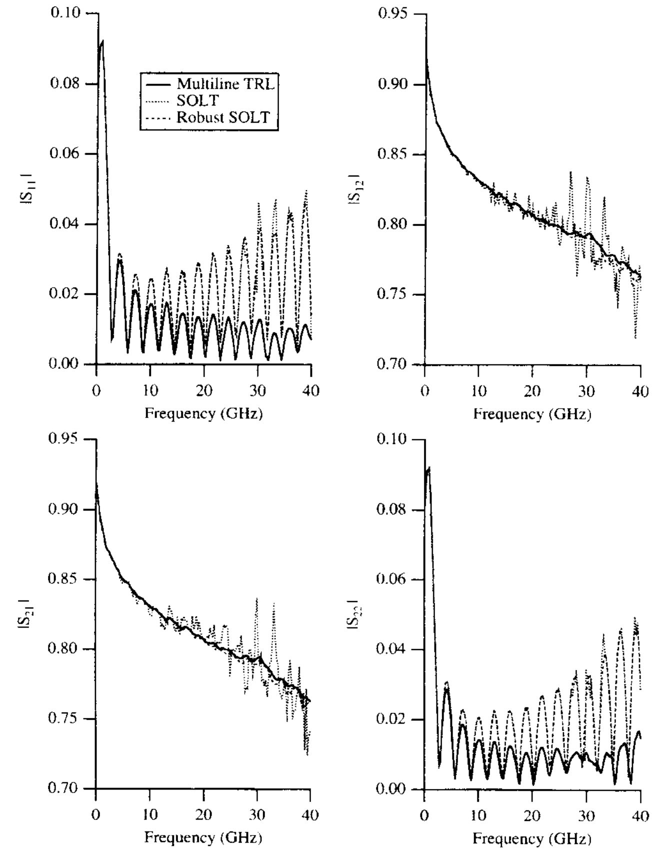

Fig. 9. Magnitude of S-parameters using multiline TRL, SOLT, and robust SOLT to measure a 19-mm coplanar-waveguide transmission line. The open standard is defined by the simplistic model.

with just slightly less accuracy than SOLT with the load measured on both ports.

Category E calibrations also ignore either the open or short, but they remeasure the reflect, rather than the load. For a simplistically modeled load, these calibrations provide good transmission accuracy and reflection accuracy comparable to SOLT. When the reflect present on both ports was simplistically modeled, however, these calibrations proved to be disastrous. Again, this is not surprising since it means that two of the three standards are simplistically modeled. Fig. 8 illustrates the effect of the simplistic open model on $O_1L_1O_2T$ and SOLT. $O_1L_1O_2T$ could not obtain any of the S-parameters with any accuracy since two of the standards were incorrectly modeled.

VII. Robust SOLT

QSOLT provides significantly better accuracy than SOLT in the transmission terms and slight improvement in the accuracy of S_{11}. However, S_{22} is very inaccurate. On the other hand, there is a simple way to get good accuracy for S_{22}: simply repeat QSOLT using the one-port standards on Port 2. The estimates of S_{12} and S_{21} turn out to be identical whether we measure the standards on Ports 1 or 2. Making use of both QSOLT calibrations, we have a new robust SOLT that provides good measurements of all four scattering parameters.

To demonstrate the effectiveness of this method, Fig. 9 compares SOLT and robust SOLT measurements to those using MultiCal [5]. We used the simplistic model of the open of Fig. 1. The robust SOLT clearly outperforms traditional SOLT. The performance of both is limited by the increasing phase of our offset open as it traverses the Smith chart.

This new robust SOLT provides greater accuracy than SOLT, but is no more difficult in terms of standards or calculations. Each calibration, however, uses *two* 12-term calibration sets, which doubles the memory requirements. It may be possible to merge the two calibration sets without loss of accuracy, but we have not found a way to do so.

VIII. Conclusions

SOLT is susceptible to significant errors in the measurement of transmission coefficients when the model of the lumped-element standards is imperfect. QSOLT provides much more accurate measurement of transmission coefficients, but offers poor accuracy for reflection coefficient on the port at which no one-port standards are measured.

The other lumped-element combinations offer a wide variety of accuracies, depending on the standards and how well they can be modeled. When choosing among these combinations, take careful consideration to avoid an inaccurate calibration. However, it is possible with these combinations to perform an accurate calibration in less time with fewer standards.

A new robust SOLT requires the same measurement and computational effort as SOLT. This robust SOLT provides much better accuracy in transmission measurements and slightly better accuracy in reflection measurements. The only drawback to the robust SOLT is the doubled memory requirements. It may be possible to merge the two calibration sets into one and thereby eliminate this minor deficiency.

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